5.3 Take Another Spin
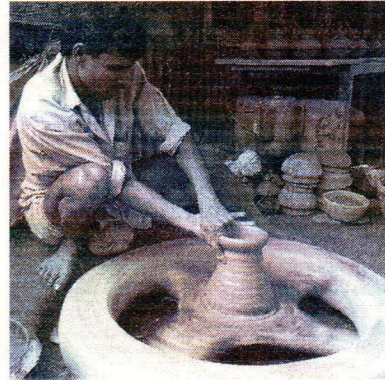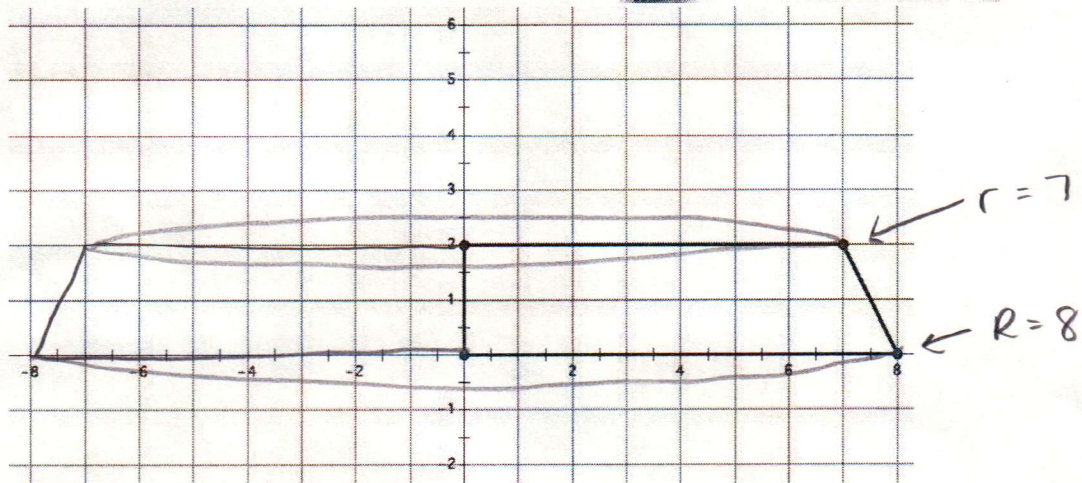
A *Solidify Understanding Task*
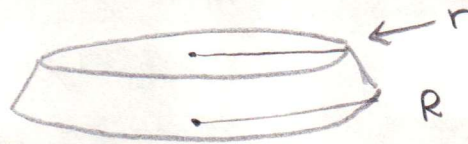
The trapezoid shown below is revolved about the *y*-axis to form a frustum (e.g., bottom slice) of a cone.

frustrum

1. Draw a sketch of the three-dimensional object formed by rotating the trapezoid about the *y*-axis.

2. Find the volume of the object formed. Explain how you used the diagram to help you find the volume.

$$V = \frac{1}{3}\pi h (R^2 + Rr + r^2)$$

$$V = \frac{1}{3}\pi (2)(8^2 + (8)(7) + 7^2) = 353.95 \text{ units}^3$$

You have made use of the formulas for cylinders and cones in your work with solids of revolution. Sometimes a solid of revolution cannot be decomposed exactly into cylinders and cones. We can approximate the volume of solids of revolution whose cross sections include curved edges by replacing them with line segments.

3. The following diagram shows the cross section of a flower vase. Approximate the volume of the vase by using line segments to approximate the curved edges. (Show the line segments you used to approximate the figure on the diagram.)

423.07

$\pi(4)(1)$
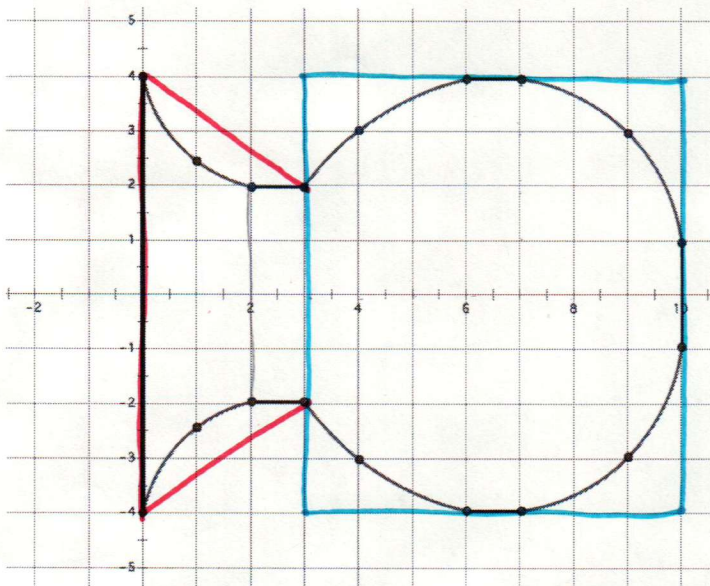
12.57

Frustrum

58.64

Cylinder

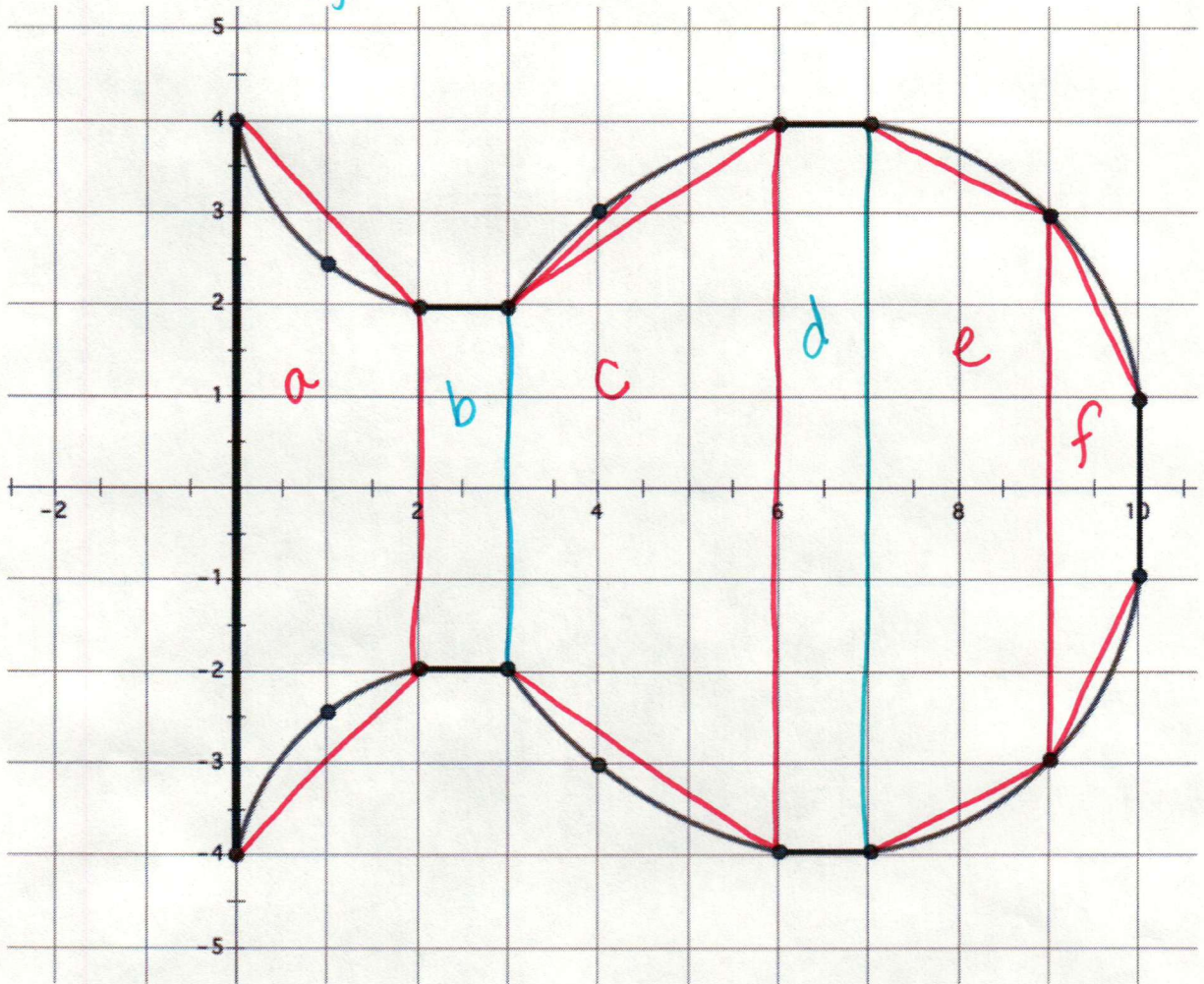$V = \pi r^2 h$

$V = \pi (4)^2 (7) = 351.86$

Frustrum

$V = \frac{1}{3} \pi h (R^2 + Rr + r^2)$

$V = \frac{1}{3} \pi (3)(4^2 + (4)(2) + 2^2)$

$= 87.96$

$V = 351.86 + 87.96 = \boxed{439.82 \text{ units}^3}$

4. Describe and carry out a strategy that will improve your approximation for the volume of the vase.

mathematics
vision project

a) $V = \frac{1}{3}\pi(2)\left(4^2 + (4)(2) + 2^2\right) = 58.64$

b) $V = \pi(2)^2(1) = 12.57$

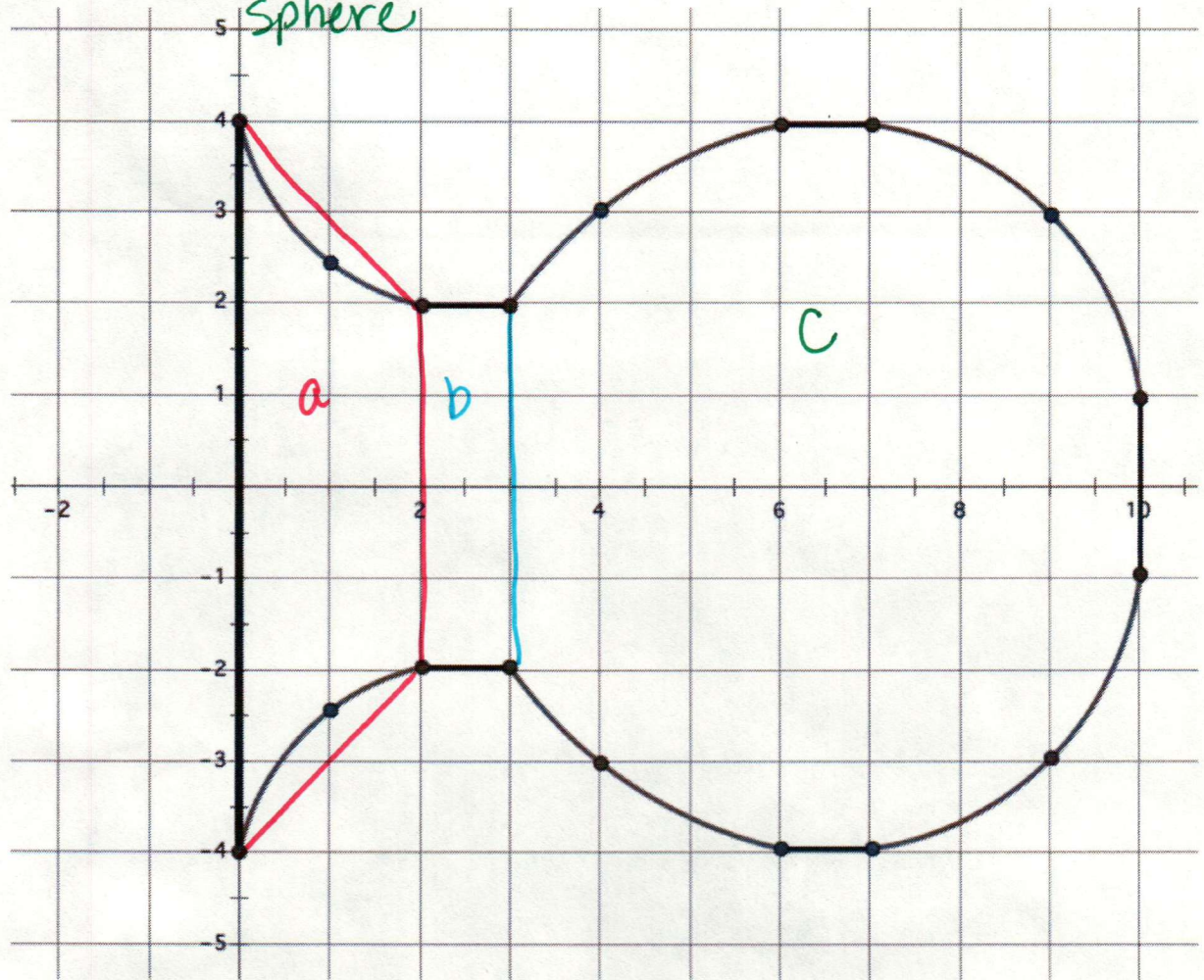c) $V = \frac{1}{3}\pi(3)\left(4^2 + (4)(2) + 2^2\right) = 87.96$

d) $V = \pi(4)^2(1) = 50.27$

e) $V = \frac{1}{3}\pi(2)\left(4^2 + (4)(3) + 3^2\right) = 77.49$

f) $V = \frac{1}{3}\pi(1)\left(3^2 + (3)(1) + 1^2\right) = 13.61$

$V = a + b + c + d + e + f = 300.54 \text{ units}^3$

frustrum
cylinder
sphere

a

b

C

a) $V = \frac{1}{3}\pi(2)(4^2 + (4)(2) + 2^2) = 58.64$

b) $V = \pi(2^2)(1) = 12.57$

c) $V = \frac{4}{3}\pi(4^3) = 268.08$

$V = a + b + c = 339.29 \text{ units}^3$